Supplementary material to the article:

Multi forests: Variable importance for multi-class outcomes

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A Simulation study

A.1 Class-specific distributions of the informative covariates

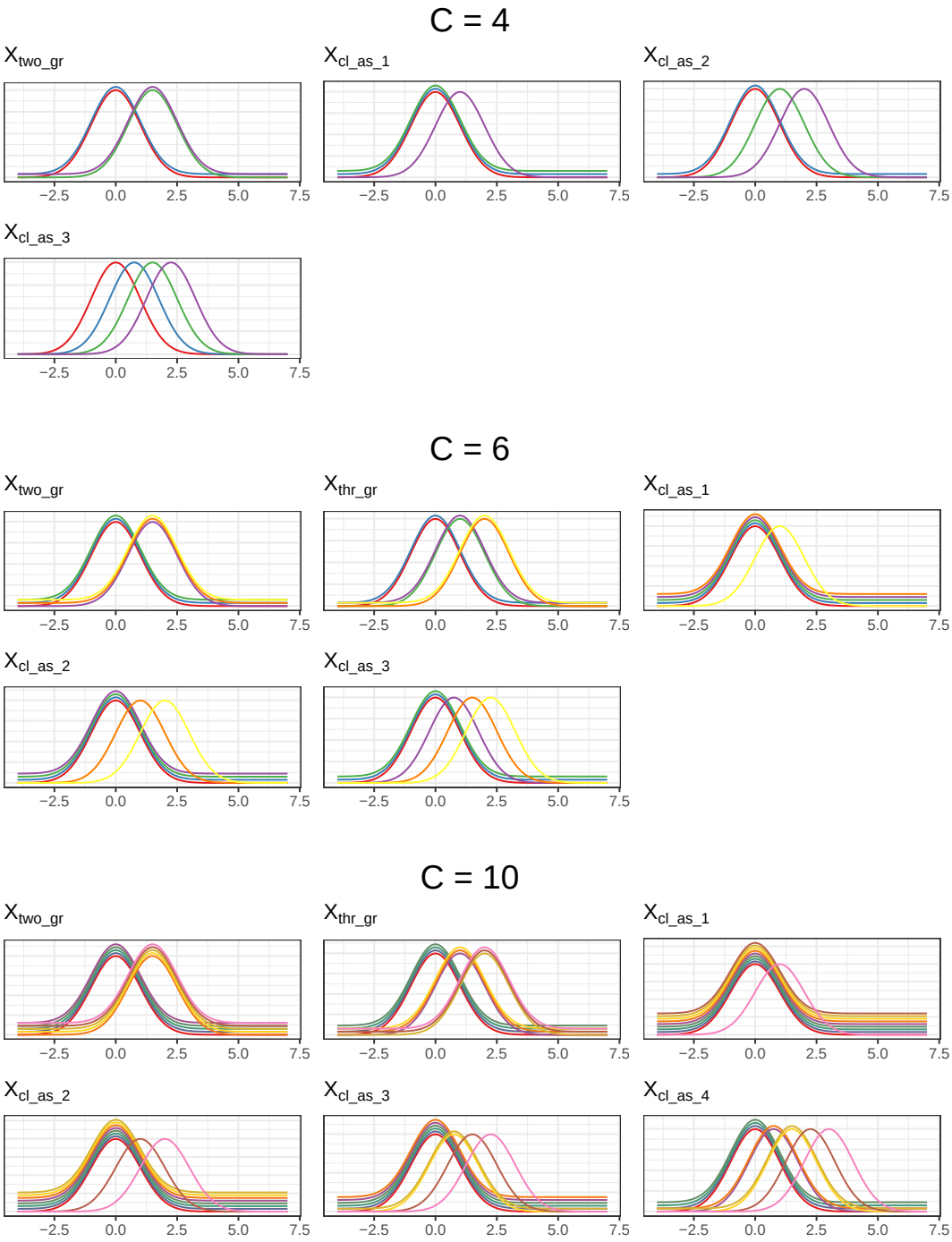

Fig. S1: Class-specific distributions of the informative covariates. Each line shows the density of a covariate within a class.

A.2 Multi-class and discriminatory VIM values

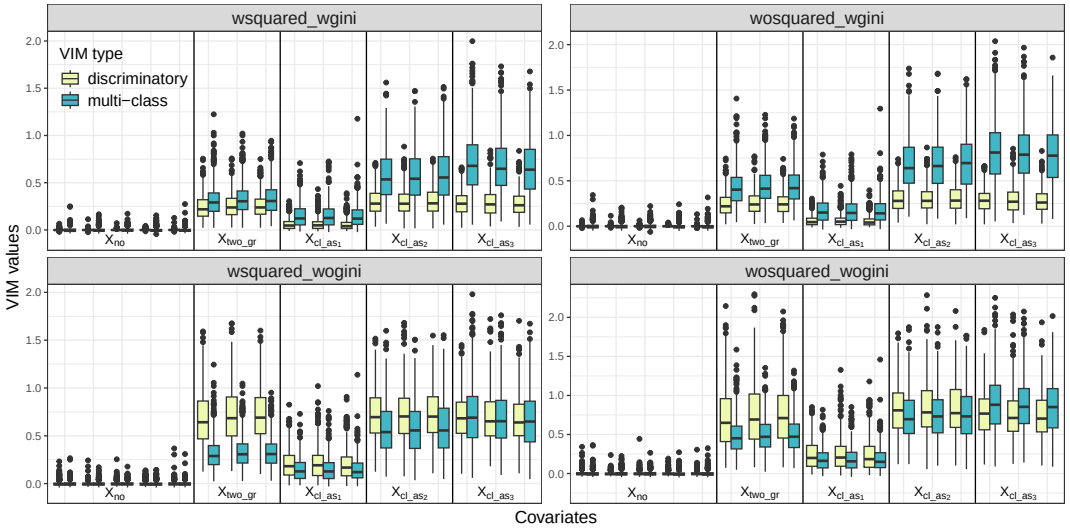

Fig. S2: Multi-class and discriminatory VIM values obtained for the four versions of multi-forest for all simulated datasets with $C = 4$ and $n = 100$. For visual clarity, the VIM values of only five of the 50 noise covariates are shown.

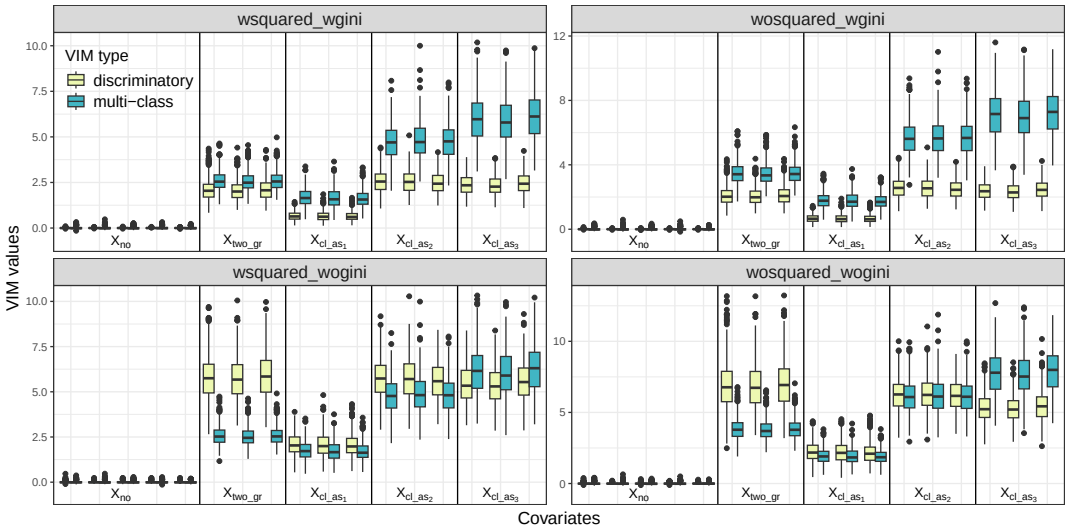

Fig. S3: Multi-class and discriminatory VIM values obtained for the four versions of multi-forest for all simulated datasets with $C = 4$ and $n = 500$. For visual clarity, the VIM values of only five of the 50 noise covariates are shown.

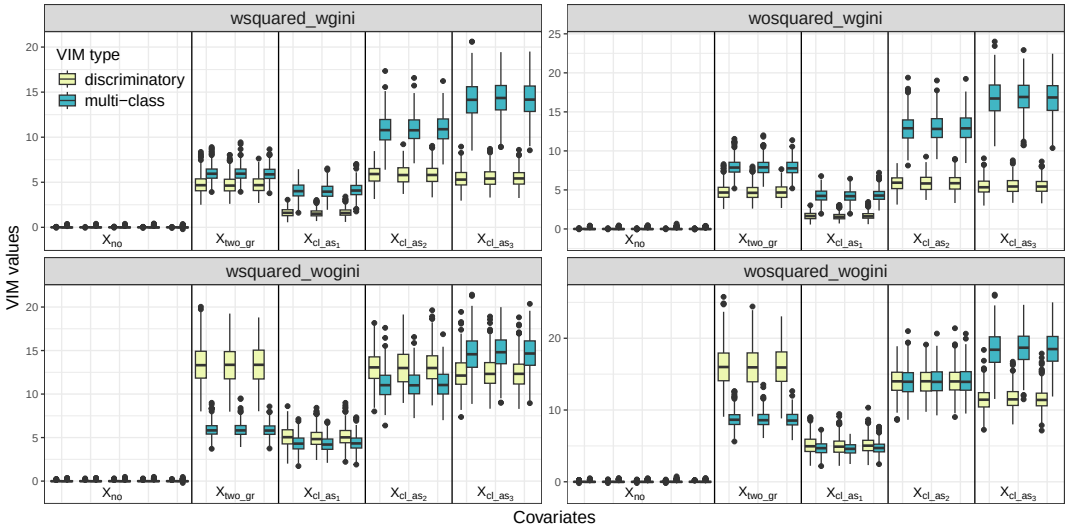

Fig. S4: Multi-class and discriminatory VIM values obtained for the four versions of multi-forest for all simulated datasets with $C = 4$ and $n = 1000$. For visual clarity, the VIM values of only five of the 50 noise covariates are shown.

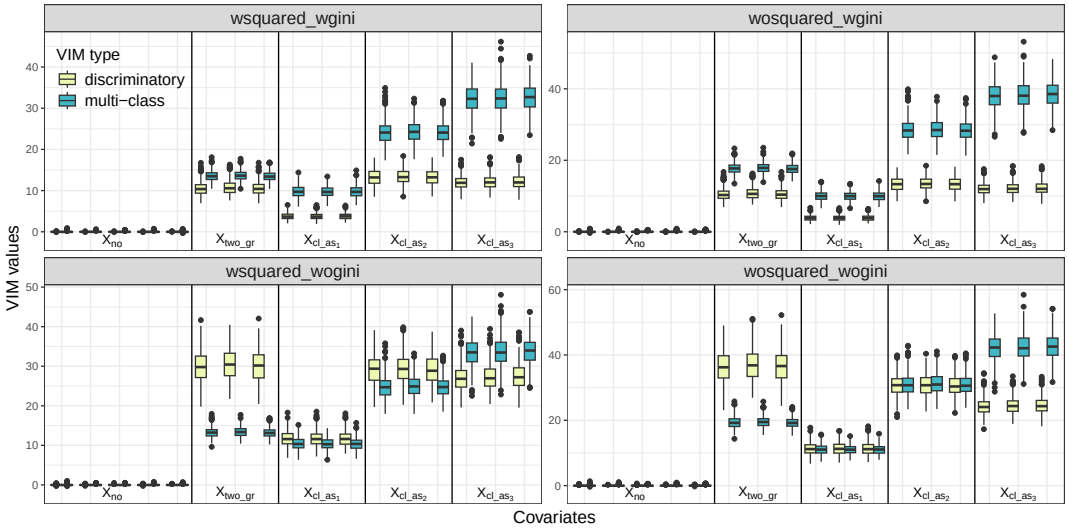

Fig. S5: Multi-class and discriminatory VIM values obtained for the four versions of multi-forest for all simulated datasets with $C = 4$ and $n = 2000$. For visual clarity, the VIM values of only five of the 50 noise covariates are shown.

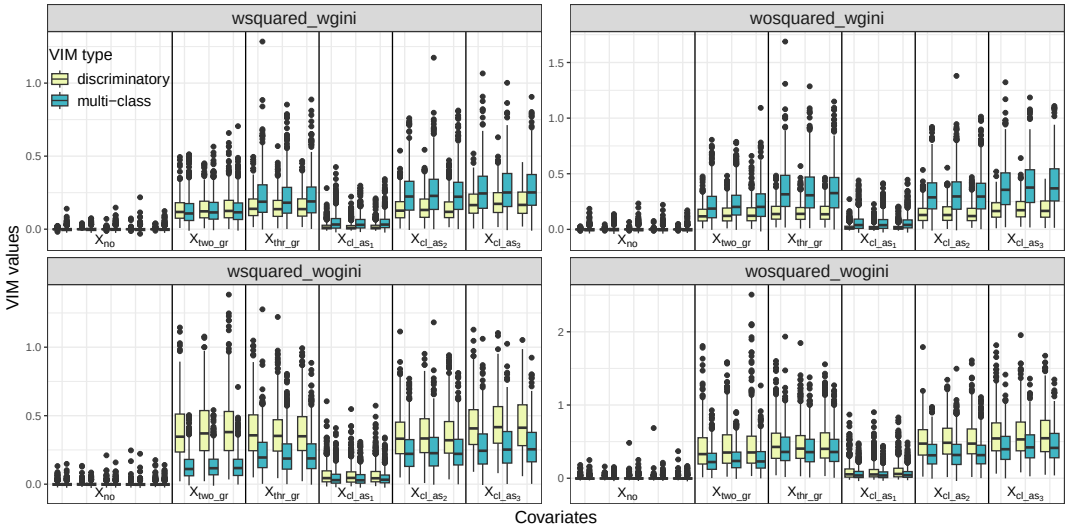

Fig. S6: Multi-class and discriminatory VIM values obtained for the four versions of multi-forest for all simulated datasets with $C = 6$ and $n = 100$. For visual clarity, the VIM values of only five of the 50 noise covariates are shown.

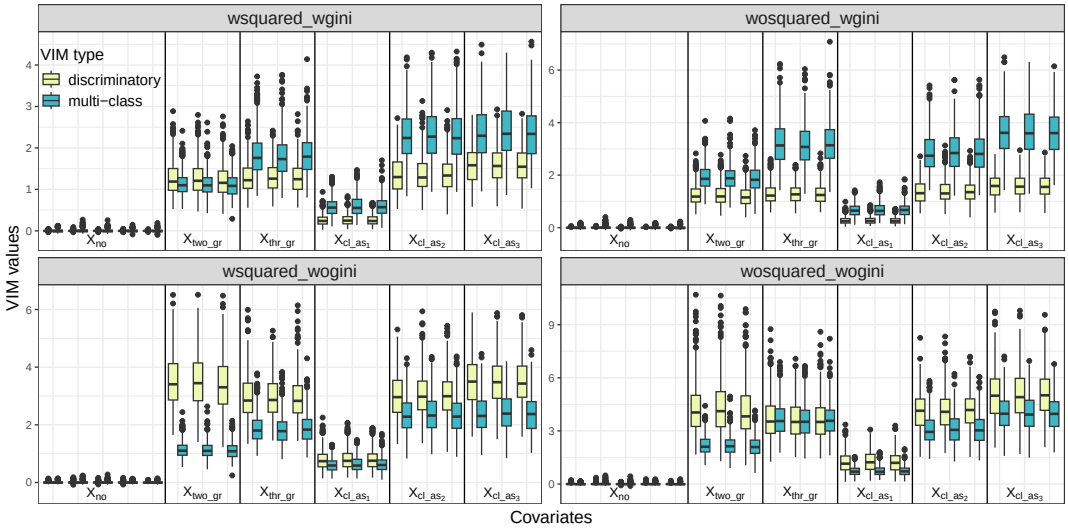

Fig. S7: Multi-class and discriminatory VIM values obtained for the four versions of multi-forest for all simulated datasets with $C = 6$ and $n = 500$. For visual clarity, the VIM values of only five of the 50 noise covariates are shown.

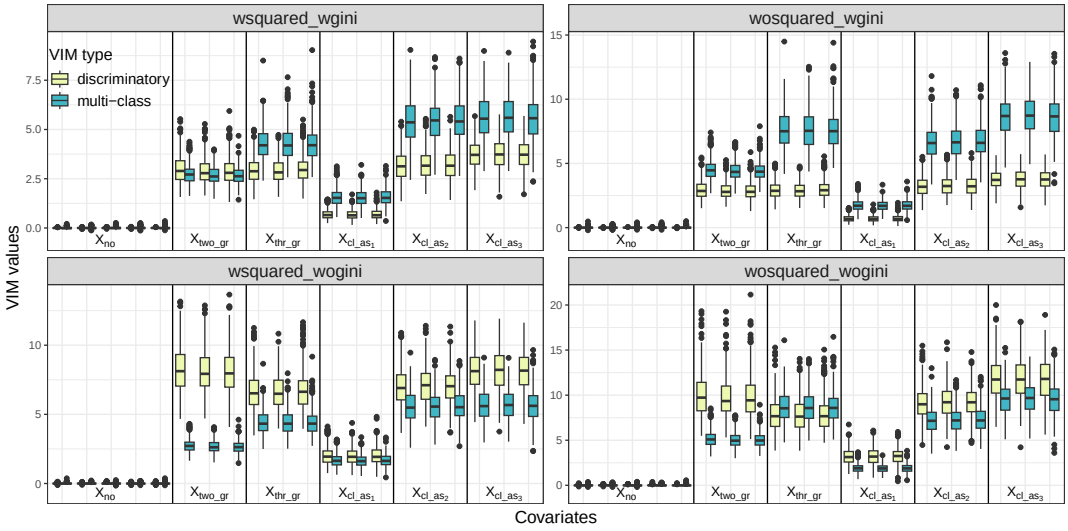

Fig. S8: Multi-class and discriminatory VIM values obtained for the four versions of multi-forest for all simulated datasets with $C = 6$ and $n = 1000$. For visual clarity, the VIM values of only five of the 50 noise covariates are shown.

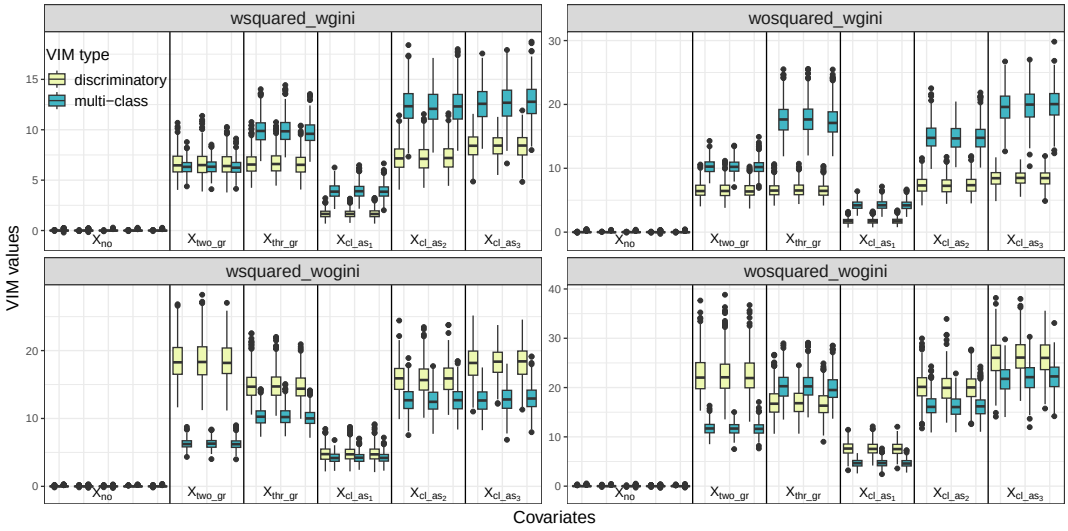

Fig. S9: Multi-class and discriminatory VIM values obtained for the four versions of multi-forest for all simulated datasets with $C = 6$ and $n = 2000$. For visual clarity, the VIM values of only five of the 50 noise covariates are shown.

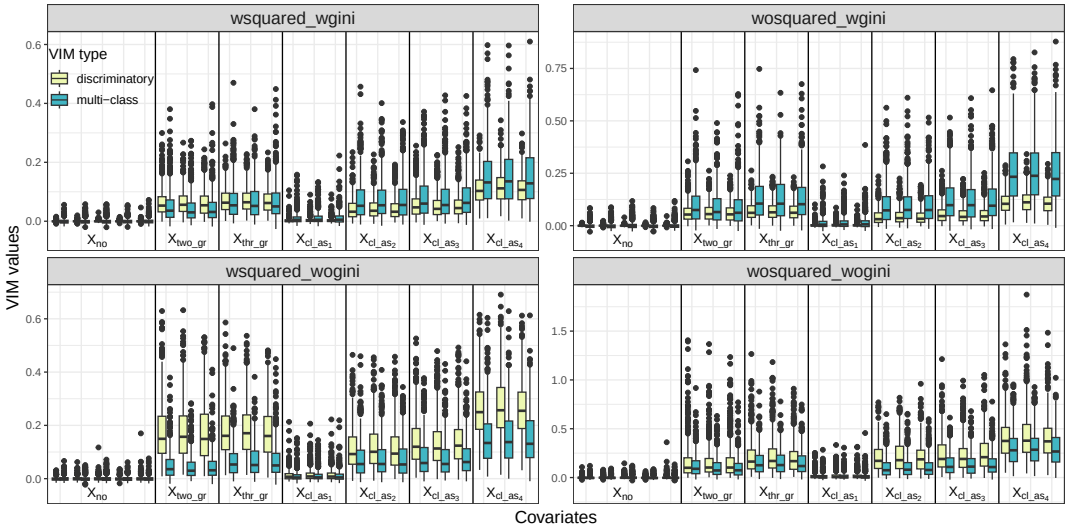

Fig. S10: Multi-class and discriminatory VIM values obtained for the four versions of multi-forest for all simulated datasets with $C = 10$ and $n = 100$. For visual clarity, the VIM values of only five of the 50 noise covariates are shown.

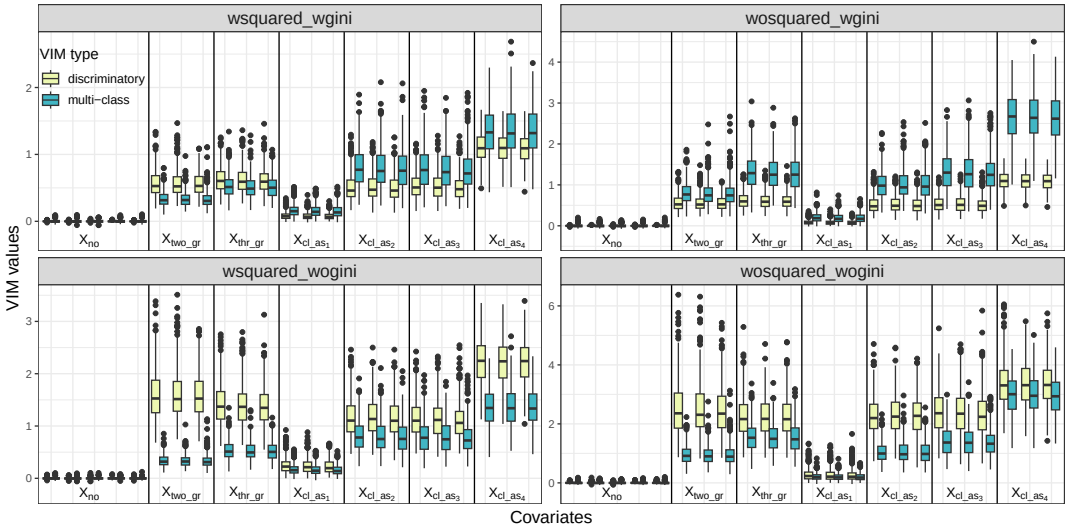

Fig. S11: Multi-class and discriminatory VIM values obtained for the four versions of multi-forest for all simulated datasets with $C = 10$ and $n = 500$. For visual clarity, the VIM values of only five of the 50 noise covariates are shown.

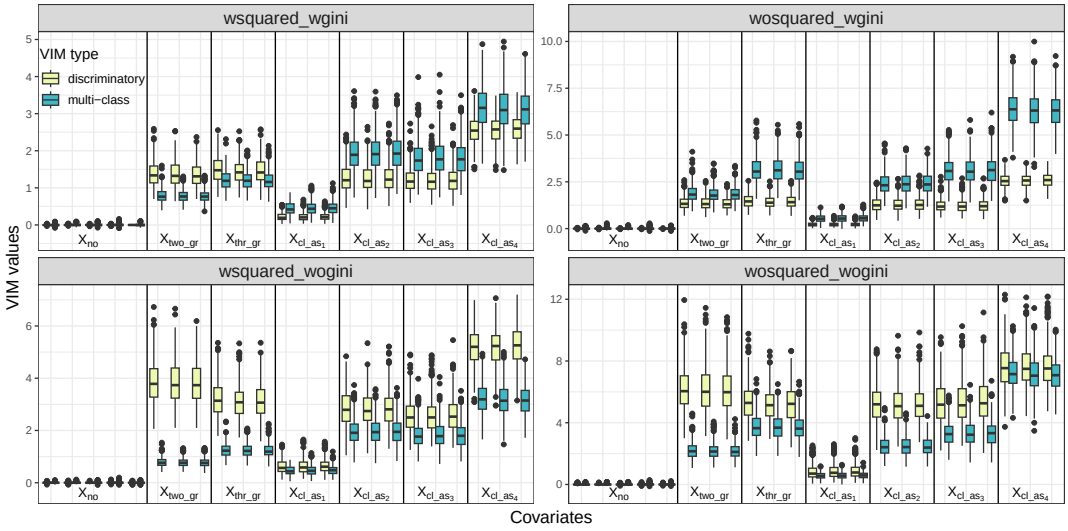

Fig. S12: Multi-class and discriminatory VIM values obtained for the four versions of multi-forest for all simulated datasets with $C = 10$ and $n = 1000$. For visual clarity, the VIM values of only five of the 50 noise covariates are shown.

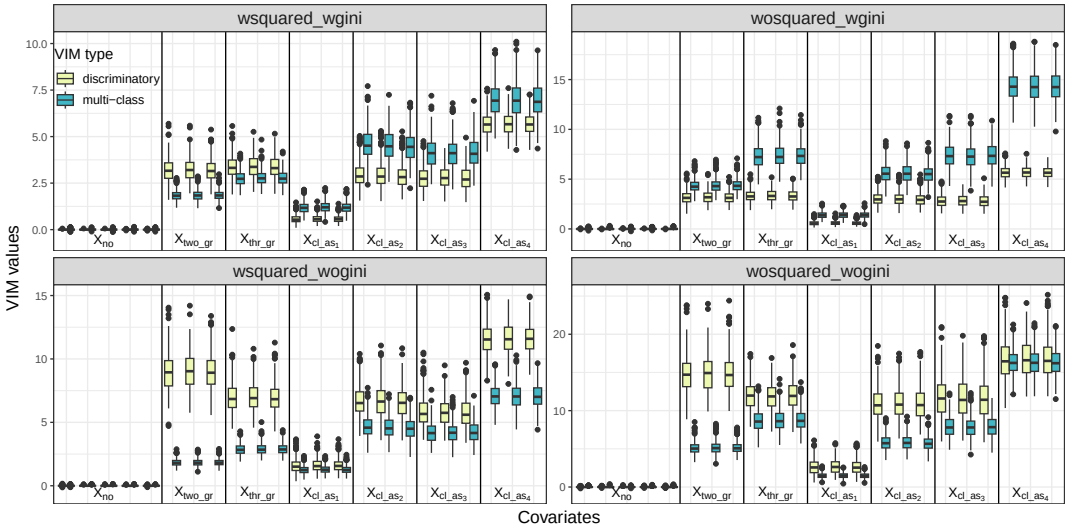

Fig. S13: Multi-class and discriminatory VIM values obtained for the four versions of multi-forest for all simulated datasets with $C = 10$ and $n = 2000$. For visual clarity, the VIM values of only five of the 50 noise covariates are shown.

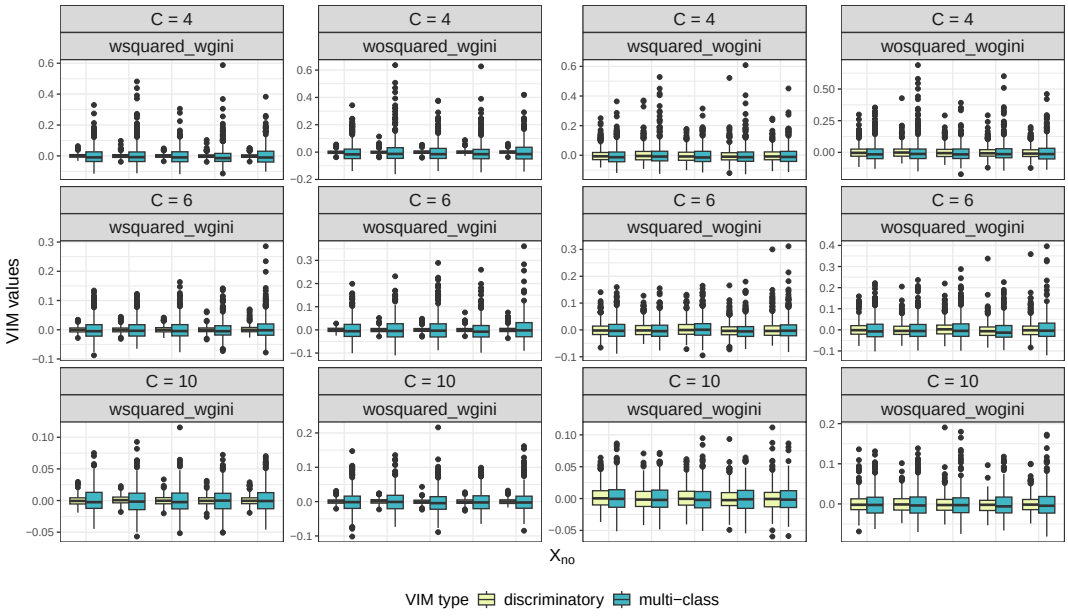

Fig. S14: Multi-class and discriminatory VIM values obtained for the four versions of multi-forest for all simulated datasets: noise covariates only. For visual clarity, the VIM values of only five of the 50 noise covariates are shown.

A.3 Mean AUC values with 95% confidence intervals

Table S1: Mean AUC values with 95% confidence intervals per considered sample size and method for $C = 4$: influential covariates vs. noise covariates.

n	method	X_{two_gr}	$X_{cl\_as\_1}$	$X_{cl\_as\_2}$	$X_{cl\_as\_3}$
100	perm	1.00 [1.00, 1.00]	0.96 [0.96, 0.97]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
100	gini_corr	1.00 [1.00, 1.00]	0.95 [0.95, 0.96]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
100	discr. VIM (wsquared_wgini)	1.00 [1.00, 1.00]	0.92 [0.91, 0.93]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
100	discr. VIM (wosquared_wgini)	1.00 [1.00, 1.00]	0.92 [0.91, 0.93]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
100	discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	0.97 [0.96, 0.97]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
100	discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	0.97 [0.96, 0.97]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
100	multi-class VIM (wsquared_wgini)	1.00 [1.00, 1.00]	0.95 [0.94, 0.95]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
100	multi-class VIM (wosquared_wgini)	1.00 [1.00, 1.00]	0.95 [0.94, 0.95]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
100	multi-class VIM (wsquared_wogini)	1.00 [1.00, 1.00]	0.95 [0.94, 0.95]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
100	multi-class VIM (wosquared_wogini)	1.00 [1.00, 1.00]	0.94 [0.94, 0.95]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	perm	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	gini_corr	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	discr. VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	discr. VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	multi-class VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	multi-class VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	multi-class VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	multi-class VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	perm	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	gini_corr	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	discr. VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	discr. VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	perm	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	gini_corr	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	discr. VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	discr. VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]

Table S2: Mean AUC values with 95% confidence intervals per considered sample size and method for $C = 6$: influential covariates vs. noise covariates.

n	method	X_{two_gr}	X_{thr_gr}	$X_{cl\_as\_1}$	$X_{cl\_as\_2}$	$X_{cl\_as\_3}$
100	perm	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.90 [0.89, 0.91]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
100	gini_corr	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.87 [0.85, 0.88]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
100	discr. VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.82 [0.81, 0.83]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
100	discr. VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.82 [0.81, 0.83]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
100	discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.90 [0.89, 0.91]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
100	discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.90 [0.89, 0.91]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
100	multi-class VIM (wsquared_wgini)	0.98 [0.98, 0.99]	1.00 [0.99, 1.00]	0.85 [0.84, 0.86]	1.00 [0.99, 1.00]	1.00 [0.99, 1.00]
100	multi-class VIM (wosquared_wgini)	0.99 [0.99, 0.99]	1.00 [1.00, 1.00]	0.84 [0.83, 0.86]	0.99 [0.99, 1.00]	1.00 [1.00, 1.00]
100	multi-class VIM (wsquared_wogini)	0.98 [0.98, 0.99]	0.99 [0.99, 1.00]	0.85 [0.84, 0.86]	0.99 [0.99, 1.00]	1.00 [0.99, 1.00]
100	multi-class VIM (wosquared_wogini)	0.99 [0.99, 0.99]	1.00 [1.00, 1.00]	0.84 [0.83, 0.86]	0.99 [0.99, 1.00]	1.00 [1.00, 1.00]
500	perm	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	gini_corr	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	discr. VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	discr. VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	multi-class VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	multi-class VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	multi-class VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	multi-class VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	perm	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	gini_corr	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	discr. VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	discr. VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	perm	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	gini_corr	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	discr. VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	discr. VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]

Table S3: Mean AUC values with 95% confidence intervals per considered sample size and method for $C = 10$: influential covariates vs. noise covariates.

n	method	X_{two_gr}	X_{thr_gr}	$X_{cl\_as\_1}$	$X_{cl\_as\_2}$	$X_{cl\_as\_3}$	$X_{cl\_as\_4}$
100	perm	0.99 [0.99, 0.99]	1.00 [1.00, 1.00]	0.78 [0.77, 0.79]	0.99 [0.98, 0.99]	0.99 [0.99, 0.99]	1.00 [1.00, 1.00]
100	gini_corr	0.99 [0.99, 0.99]	1.00 [1.00, 1.00]	0.73 [0.71, 0.74]	0.97 [0.97, 0.98]	0.99 [0.99, 0.99]	1.00 [1.00, 1.00]
100	discr. VIM (wsquared_wgini)	0.99 [0.99, 1.00]	1.00 [1.00, 1.00]	0.66 [0.64, 0.68]	0.98 [0.97, 0.98]	0.99 [0.99, 0.99]	1.00 [1.00, 1.00]
100	discr. VIM (wosquared_wgini)	1.00 [0.99, 1.00]	1.00 [1.00, 1.00]	0.65 [0.63, 0.66]	0.98 [0.97, 0.98]	0.99 [0.99, 0.99]	1.00 [1.00, 1.00]
100	discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.76 [0.75, 0.78]	0.99 [0.99, 0.99]	1.00 [0.99, 1.00]	1.00 [1.00, 1.00]
100	discr. VIM (wosquared_wogini)	0.99 [0.98, 0.99]	1.00 [0.99, 1.00]	0.74 [0.72, 0.75]	0.99 [0.98, 0.99]	0.99 [0.99, 1.00]	1.00 [1.00, 1.00]
100	multi-class VIM (wsquared_wgini)	0.91 [0.90, 0.92]	0.95 [0.95, 0.96]	0.71 [0.69, 0.72]	0.95 [0.95, 0.96]	0.96 [0.96, 0.97]	1.00 [0.99, 1.00]
100	multi-class VIM (wosquared_wgini)	0.93 [0.92, 0.94]	0.96 [0.95, 0.97]	0.69 [0.67, 0.70]	0.95 [0.94, 0.95]	0.96 [0.96, 0.97]	1.00 [1.00, 1.00]
100	multi-class VIM (wsquared_wogini)	0.91 [0.90, 0.92]	0.95 [0.95, 0.96]	0.71 [0.70, 0.72]	0.95 [0.95, 0.96]	0.96 [0.95, 0.97]	0.99 [0.99, 1.00]
100	multi-class VIM (wosquared_wogini)	0.94 [0.94, 0.95]	0.97 [0.96, 0.97]	0.67 [0.66, 0.69]	0.94 [0.94, 0.95]	0.96 [0.96, 0.97]	1.00 [1.00, 1.00]
500	perm	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.99 [0.99, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	gini_corr	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.99 [0.98, 0.99]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	discr. VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.99 [0.98, 0.99]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	discr. VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.99 [0.99, 0.99]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.99 [0.99, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	multi-class VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.99 [0.99, 0.99]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	multi-class VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.98 [0.98, 0.99]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	multi-class VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.99 [0.99, 0.99]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
500	multi-class VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.98 [0.98, 0.99]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	perm	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	gini_corr	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	discr. VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	discr. VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	perm	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	gini_corr	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	discr. VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	discr. VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wosquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]

Table S4: Mean AUC values with 95% confidence intervals per considered sample size and method for $C = 4$: class-associated covariates vs. X_{two_gr}.

n	method	$X_{cl\_as\_1}$	$X_{cl\_as\_2}$	$X_{cl\_as\_3}$
100	perm	0.11 [0.09, 0.12]	0.62 [0.60, 0.64]	0.63 [0.61, 0.65]
100	gini_corr	0.06 [0.05, 0.07]	0.62 [0.60, 0.64]	0.66 [0.64, 0.68]
100	discr. VIM (wsquared_wgini)	0.06 [0.05, 0.07]	0.60 [0.58, 0.63]	0.57 [0.55, 0.59]
100	discr. VIM (wosquared_wgini)	0.06 [0.05, 0.07]	0.61 [0.59, 0.63]	0.58 [0.55, 0.60]
100	discr. VIM (wsquared_wogini)	0.06 [0.05, 0.07]	0.53 [0.51, 0.55]	0.49 [0.47, 0.51]
100	discr. VIM (wosquared_wogini)	0.12 [0.11, 0.14]	0.58 [0.56, 0.60]	0.54 [0.51, 0.56]
100	multi-class VIM (wsquared_wgini)	0.19 [0.17, 0.20]	0.79 [0.77, 0.81]	0.85 [0.83, 0.86]
100	multi-class VIM (wosquared_wgini)	0.13 [0.12, 0.14]	0.75 [0.73, 0.77]	0.83 [0.81, 0.84]
100	multi-class VIM (wsquared_wogini)	0.19 [0.18, 0.21]	0.78 [0.77, 0.80]	0.85 [0.83, 0.86]
100	multi-class VIM (wosquared_wogini)	0.12 [0.10, 0.13]	0.74 [0.72, 0.76]	0.81 [0.80, 0.83]
100	multi-class VIM - discr. VIM (wsquared_wgini)	0.58 [0.56, 0.60]	0.79 [0.78, 0.81]	0.88 [0.87, 0.89]
100	multi-class VIM - discr. VIM (wosquared_wgini)	0.35 [0.33, 0.37]	0.77 [0.75, 0.78]	0.86 [0.85, 0.88]
100	multi-class VIM - discr. VIM (wsquared_wogini)	0.92 [0.91, 0.93]	0.78 [0.76, 0.80]	0.87 [0.86, 0.89]
100	multi-class VIM - discr. VIM (wosquared_wogini)	0.70 [0.68, 0.72]	0.66 [0.64, 0.69]	0.84 [0.83, 0.86]
500	perm	0.01 [0.01, 0.01]	0.73 [0.72, 0.75]	0.80 [0.78, 0.82]
500	gini_corr	0.00 [0.00, 0.00]	0.73 [0.71, 0.74]	0.80 [0.78, 0.81]
500	discr. VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.70 [0.68, 0.72]	0.65 [0.63, 0.67]
500	discr. VIM (wosquared_wgini)	0.00 [0.00, 0.01]	0.71 [0.70, 0.73]	0.66 [0.64, 0.68]
500	discr. VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.49 [0.47, 0.51]	0.42 [0.40, 0.44]
500	discr. VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.38 [0.36, 0.40]	0.21 [0.19, 0.23]
500	multi-class VIM (wsquared_wgini)	0.09 [0.08, 0.10]	0.98 [0.98, 0.98]	0.99 [0.99, 1.00]
500	multi-class VIM (wosquared_wgini)	0.01 [0.01, 0.02]	0.96 [0.95, 0.97]	0.99 [0.99, 0.99]
500	multi-class VIM (wsquared_wogini)	0.13 [0.12, 0.14]	0.98 [0.98, 0.99]	1.00 [0.99, 1.00]
500	multi-class VIM (wosquared_wogini)	0.01 [0.01, 0.02]	0.95 [0.95, 0.96]	0.99 [0.99, 0.99]
500	multi-class VIM - discr. VIM (wsquared_wgini)	0.85 [0.83, 0.86]	0.99 [0.98, 0.99]	1.00 [1.00, 1.00]
500	multi-class VIM - discr. VIM (wosquared_wgini)	0.32 [0.30, 0.34]	0.97 [0.97, 0.98]	1.00 [1.00, 1.00]
500	multi-class VIM - discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	0.97 [0.96, 0.97]	1.00 [0.99, 1.00]
500	multi-class VIM - discr. VIM (wosquared_wogini)	0.99 [0.99, 0.99]	0.98 [0.98, 0.99]	1.00 [1.00, 1.00]
1000	perm	0.00 [0.00, 0.00]	0.81 [0.79, 0.82]	0.89 [0.88, 0.90]
1000	gini_corr	0.00 [0.00, 0.00]	0.79 [0.77, 0.81]	0.88 [0.87, 0.89]
1000	discr. VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.76 [0.74, 0.78]	0.69 [0.67, 0.71]
1000	discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.77 [0.75, 0.79]	0.70 [0.68, 0.72]
1000	discr. VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.46 [0.44, 0.48]	0.37 [0.35, 0.39]
1000	discr. VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.28 [0.26, 0.30]	0.08 [0.07, 0.09]
1000	multi-class VIM (wsquared_wgini)	0.05 [0.04, 0.05]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.99 [0.99, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wsquared_wogini)	0.09 [0.08, 0.10]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.99 [0.99, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM - discr. VIM (wsquared_wgini)	0.94 [0.93, 0.95]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM - discr. VIM (wosquared_wgini)	0.27 [0.25, 0.29]	0.99 [0.99, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM - discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	0.99 [0.99, 0.99]	1.00 [1.00, 1.00]
1000	multi-class VIM - discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	perm	0.00 [0.00, 0.00]	0.89 [0.88, 0.91]	0.96 [0.95, 0.97]
2000	gini_corr	0.00 [0.00, 0.00]	0.87 [0.86, 0.88]	0.95 [0.94, 0.96]
2000	discr. VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.85 [0.83, 0.86]	0.74 [0.72, 0.76]
2000	discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.86 [0.84, 0.87]	0.75 [0.73, 0.77]
2000	discr. VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.44 [0.42, 0.46]	0.29 [0.27, 0.31]
2000	discr. VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.16 [0.15, 0.18]	0.01 [0.01, 0.02]
2000	multi-class VIM (wsquared_wgini)	0.02 [0.01, 0.02]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wosquared_wgini)	0.00 [0.00, 0.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wsquared_wogini)	0.06 [0.05, 0.07]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wosquared_wogini)	0.00 [0.00, 0.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM - discr. VIM (wsquared_wgini)	0.98 [0.98, 0.99]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM - discr. VIM (wosquared_wgini)	0.22 [0.21, 0.24]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM - discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM - discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]

Table S5: Mean AUC values with 95% confidence intervals per considered sample size and method for $C = 6$: class-associated covariates vs. X_{two_gr}.

n	method	$X_{cl\_as\_1}$	$X_{cl\_as\_2}$	$X_{cl\_as\_3}$
100	perm	0.07 [0.06, 0.09]	0.51 [0.49, 0.53]	0.59 [0.57, 0.61]
100	gini_corr	0.04 [0.03, 0.05]	0.48 [0.45, 0.50]	0.60 [0.58, 0.63]
100	discr. VIM (wsquared_wgini)	0.04 [0.04, 0.05]	0.50 [0.48, 0.53]	0.63 [0.61, 0.65]
100	discr. VIM (wosquared_wgini)	0.05 [0.04, 0.05]	0.51 [0.49, 0.53]	0.63 [0.61, 0.65]
100	discr. VIM (wsquared_wogini)	0.04 [0.03, 0.04]	0.44 [0.42, 0.47]	0.57 [0.54, 0.59]
100	discr. VIM (wosquared_wogini)	0.11 [0.09, 0.12]	0.62 [0.60, 0.64]	0.68 [0.66, 0.70]
100	multi-class VIM (wsquared_wgini)	0.19 [0.17, 0.21]	0.73 [0.72, 0.75]	0.78 [0.76, 0.80]
100	multi-class VIM (wosquared_wgini)	0.13 [0.11, 0.14]	0.66 [0.63, 0.68]	0.75 [0.73, 0.77]
100	multi-class VIM (wsquared_wogini)	0.19 [0.17, 0.21]	0.73 [0.71, 0.75]	0.78 [0.76, 0.79]
100	multi-class VIM (wosquared_wogini)	0.11 [0.10, 0.12]	0.63 [0.60, 0.65]	0.74 [0.72, 0.76]
100	multi-class VIM - discr. VIM (wsquared_wgini)	0.68 [0.65, 0.70]	0.78 [0.77, 0.80]	0.72 [0.70, 0.74]
100	multi-class VIM - discr. VIM (wosquared_wgini)	0.40 [0.38, 0.42]	0.69 [0.67, 0.71]	0.73 [0.71, 0.75]
100	multi-class VIM - discr. VIM (wsquared_wogini)	0.94 [0.93, 0.95]	0.76 [0.74, 0.78]	0.66 [0.64, 0.68]
100	multi-class VIM - discr. VIM (wosquared_wogini)	0.67 [0.65, 0.69]	0.46 [0.44, 0.48]	0.50 [0.48, 0.52]
500	perm	0.00 [0.00, 0.00]	0.54 [0.52, 0.56]	0.62 [0.60, 0.64]
500	gini_corr	0.00 [0.00, 0.00]	0.45 [0.43, 0.47]	0.63 [0.60, 0.65]
500	discr. VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.58 [0.56, 0.60]	0.72 [0.70, 0.74]
500	discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.60 [0.58, 0.62]	0.73 [0.71, 0.75]
500	discr. VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.36 [0.34, 0.38]	0.53 [0.51, 0.55]
500	discr. VIM (wosquared_wogini)	0.01 [0.01, 0.01]	0.51 [0.49, 0.53]	0.67 [0.65, 0.69]
500	multi-class VIM (wsquared_wgini)	0.08 [0.06, 0.09]	0.97 [0.96, 0.98]	0.97 [0.96, 0.98]
500	multi-class VIM (wosquared_wgini)	0.01 [0.01, 0.01]	0.86 [0.85, 0.88]	0.95 [0.94, 0.96]
500	multi-class VIM (wsquared_wogini)	0.09 [0.08, 0.11]	0.97 [0.97, 0.98]	0.97 [0.96, 0.98]
500	multi-class VIM (wosquared_wogini)	0.01 [0.00, 0.01]	0.82 [0.80, 0.84]	0.94 [0.93, 0.95]
500	multi-class VIM - discr. VIM (wsquared_wgini)	0.91 [0.90, 0.93]	0.97 [0.97, 0.98]	0.92 [0.91, 0.93]
500	multi-class VIM - discr. VIM (wosquared_wgini)	0.27 [0.25, 0.29]	0.89 [0.87, 0.90]	0.94 [0.94, 0.95]
500	multi-class VIM - discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	0.96 [0.96, 0.97]	0.87 [0.86, 0.89]
500	multi-class VIM - discr. VIM (wosquared_wogini)	0.91 [0.90, 0.93]	0.76 [0.74, 0.77]	0.72 [0.70, 0.74]
1000	perm	0.00 [0.00, 0.00]	0.55 [0.53, 0.56]	0.65 [0.63, 0.66]
1000	gini_corr	0.00 [0.00, 0.00]	0.43 [0.41, 0.45]	0.65 [0.63, 0.67]
1000	discr. VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.62 [0.60, 0.64]	0.80 [0.78, 0.82]
1000	discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.64 [0.62, 0.66]	0.81 [0.79, 0.82]
1000	discr. VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.32 [0.30, 0.33]	0.53 [0.50, 0.55]
1000	discr. VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.44 [0.42, 0.46]	0.73 [0.72, 0.75]
1000	multi-class VIM (wsquared_wgini)	0.03 [0.03, 0.04]	0.99 [0.99, 1.00]	1.00 [0.99, 1.00]
1000	multi-class VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.94 [0.93, 0.95]	0.99 [0.99, 1.00]
1000	multi-class VIM (wsquared_wogini)	0.05 [0.04, 0.06]	1.00 [0.99, 1.00]	1.00 [0.99, 1.00]
1000	multi-class VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.91 [0.90, 0.92]	0.99 [0.99, 0.99]
1000	multi-class VIM - discr. VIM (wsquared_wgini)	0.98 [0.98, 0.99]	1.00 [0.99, 1.00]	0.98 [0.97, 0.98]
1000	multi-class VIM - discr. VIM (wosquared_wgini)	0.21 [0.19, 0.22]	0.95 [0.95, 0.96]	0.99 [0.99, 1.00]
1000	multi-class VIM - discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	0.99 [0.99, 0.99]	0.94 [0.93, 0.95]
1000	multi-class VIM - discr. VIM (wosquared_wogini)	0.98 [0.98, 0.99]	0.89 [0.88, 0.90]	0.81 [0.79, 0.82]
2000	perm	0.00 [0.00, 0.00]	0.57 [0.56, 0.59]	0.66 [0.64, 0.67]
2000	gini_corr	0.00 [0.00, 0.00]	0.44 [0.42, 0.45]	0.67 [0.66, 0.69]
2000	discr. VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.64 [0.62, 0.66]	0.84 [0.83, 0.86]
2000	discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.68 [0.66, 0.69]	0.86 [0.84, 0.87]
2000	discr. VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.24 [0.22, 0.25]	0.49 [0.47, 0.51]
2000	discr. VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.30 [0.29, 0.32]	0.75 [0.74, 0.77]
2000	multi-class VIM (wsquared_wgini)	0.01 [0.01, 0.01]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.99 [0.98, 0.99]	1.00 [1.00, 1.00]
2000	multi-class VIM (wsquared_wogini)	0.02 [0.02, 0.03]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.97 [0.97, 0.98]	1.00 [1.00, 1.00]
2000	multi-class VIM - discr. VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [0.99, 1.00]
2000	multi-class VIM - discr. VIM (wosquared_wgini)	0.12 [0.11, 0.13]	0.98 [0.98, 0.99]	1.00 [1.00, 1.00]
2000	multi-class VIM - discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.99 [0.99, 0.99]
2000	multi-class VIM - discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	0.97 [0.96, 0.97]	0.93 [0.92, 0.94]

Table S6: Mean AUC values with 95% confidence intervals per considered sample size and method for $C = 6$: class-associated covariates vs. X_{thr_gr}.

n	method	$X_{cl\_as\_1}$	$X_{cl\_as\_2}$	$X_{cl\_as\_3}$
100	perm	0.06 [0.05, 0.07]	0.45 [0.42, 0.47]	0.53 [0.51, 0.55]
100	gini_corr	0.03 [0.02, 0.03]	0.38 [0.36, 0.40]	0.51 [0.49, 0.53]
100	discr. VIM (wsquared_wgini)	0.04 [0.03, 0.04]	0.45 [0.43, 0.48]	0.59 [0.57, 0.61]
100	discr. VIM (wosquared_wgini)	0.04 [0.03, 0.04]	0.46 [0.44, 0.48]	0.59 [0.57, 0.61]
100	discr. VIM (wsquared_wogini)	0.03 [0.03, 0.04]	0.45 [0.43, 0.48]	0.58 [0.56, 0.61]
100	discr. VIM (wosquared_wogini)	0.06 [0.05, 0.07]	0.57 [0.55, 0.59]	0.63 [0.61, 0.66]
100	multi-class VIM (wsquared_wgini)	0.09 [0.08, 0.11]	0.56 [0.54, 0.58]	0.60 [0.58, 0.62]
100	multi-class VIM (wosquared_wgini)	0.06 [0.05, 0.07]	0.47 [0.44, 0.49]	0.57 [0.55, 0.60]
100	multi-class VIM (wsquared_wogini)	0.09 [0.08, 0.11]	0.55 [0.53, 0.58]	0.60 [0.57, 0.62]
100	multi-class VIM (wosquared_wogini)	0.05 [0.04, 0.06]	0.43 [0.41, 0.46]	0.56 [0.54, 0.58]
100	multi-class VIM - discr. VIM (wsquared_wgini)	0.43 [0.40, 0.45]	0.61 [0.59, 0.63]	0.56 [0.54, 0.58]
100	multi-class VIM - discr. VIM (wosquared_wgini)	0.18 [0.16, 0.20]	0.48 [0.45, 0.50]	0.53 [0.51, 0.56]
100	multi-class VIM - discr. VIM (wsquared_wogini)	0.83 [0.82, 0.85]	0.61 [0.59, 0.63]	0.52 [0.49, 0.54]
100	multi-class VIM - discr. VIM (wosquared_wogini)	0.56 [0.53, 0.58]	0.35 [0.33, 0.37]	0.41 [0.39, 0.43]
500	perm	0.00 [0.00, 0.00]	0.43 [0.41, 0.45]	0.52 [0.50, 0.54]
500	gini_corr	0.00 [0.00, 0.00]	0.28 [0.26, 0.30]	0.44 [0.42, 0.46]
500	discr. VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.55 [0.53, 0.57]	0.70 [0.68, 0.72]
500	discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.56 [0.54, 0.58]	0.70 [0.68, 0.72]
500	discr. VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.54 [0.52, 0.56]	0.69 [0.67, 0.71]
500	discr. VIM (wosquared_wogini)	0.02 [0.01, 0.02]	0.64 [0.62, 0.66]	0.78 [0.76, 0.80]
500	multi-class VIM (wsquared_wgini)	0.01 [0.00, 0.01]	0.73 [0.71, 0.75]	0.75 [0.73, 0.77]
500	multi-class VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.39 [0.37, 0.41]	0.64 [0.62, 0.66]
500	multi-class VIM (wsquared_wogini)	0.01 [0.00, 0.01]	0.73 [0.71, 0.75]	0.74 [0.72, 0.76]
500	multi-class VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.34 [0.32, 0.36]	0.61 [0.59, 0.63]
500	multi-class VIM - discr. VIM (wsquared_wgini)	0.35 [0.33, 0.37]	0.74 [0.72, 0.76]	0.64 [0.62, 0.66]
500	multi-class VIM - discr. VIM (wosquared_wgini)	0.00 [0.00, 0.01]	0.33 [0.31, 0.36]	0.56 [0.54, 0.59]
500	multi-class VIM - discr. VIM (wsquared_wogini)	0.94 [0.93, 0.95]	0.69 [0.67, 0.71]	0.50 [0.48, 0.53]
500	multi-class VIM - discr. VIM (wosquared_wogini)	0.28 [0.26, 0.30]	0.19 [0.17, 0.20]	0.26 [0.24, 0.28]
1000	perm	0.00 [0.00, 0.00]	0.43 [0.41, 0.45]	0.53 [0.51, 0.55]
1000	gini_corr	0.00 [0.00, 0.00]	0.21 [0.20, 0.23]	0.41 [0.39, 0.43]
1000	discr. VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.60 [0.58, 0.62]	0.79 [0.77, 0.81]
1000	discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.63 [0.61, 0.64]	0.80 [0.78, 0.82]
1000	discr. VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.59 [0.57, 0.61]	0.78 [0.77, 0.80]
1000	discr. VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.71 [0.69, 0.73]	0.90 [0.89, 0.91]
1000	multi-class VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.81 [0.79, 0.82]	0.82 [0.80, 0.83]
1000	multi-class VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.33 [0.31, 0.36]	0.68 [0.66, 0.70]
1000	multi-class VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.80 [0.79, 0.82]	0.80 [0.79, 0.82]
1000	multi-class VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.27 [0.25, 0.29]	0.65 [0.63, 0.67]
1000	multi-class VIM - discr. VIM (wsquared_wgini)	0.27 [0.25, 0.29]	0.80 [0.78, 0.82]	0.67 [0.65, 0.69]
1000	multi-class VIM - discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.23 [0.21, 0.25]	0.56 [0.54, 0.58]
1000	multi-class VIM - discr. VIM (wsquared_wogini)	0.98 [0.97, 0.99]	0.71 [0.69, 0.72]	0.44 [0.42, 0.46]
1000	multi-class VIM - discr. VIM (wosquared_wogini)	0.08 [0.07, 0.09]	0.09 [0.08, 0.10]	0.13 [0.12, 0.15]
2000	perm	0.00 [0.00, 0.00]	0.41 [0.39, 0.42]	0.48 [0.46, 0.50]
2000	gini_corr	0.00 [0.00, 0.00]	0.14 [0.13, 0.16]	0.32 [0.30, 0.34]
2000	discr. VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.62 [0.60, 0.65]	0.85 [0.84, 0.87]
2000	discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.67 [0.65, 0.68]	0.86 [0.85, 0.88]
2000	discr. VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.63 [0.61, 0.65]	0.85 [0.84, 0.87]
2000	discr. VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.78 [0.77, 0.80]	0.97 [0.97, 0.98]
2000	multi-class VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.87 [0.86, 0.88]	0.90 [0.89, 0.91]
2000	multi-class VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.20 [0.19, 0.22]	0.74 [0.72, 0.76]
2000	multi-class VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.86 [0.85, 0.87]	0.87 [0.86, 0.89]
2000	multi-class VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.14 [0.12, 0.15]	0.68 [0.67, 0.70]
2000	multi-class VIM - discr. VIM (wsquared_wgini)	0.20 [0.18, 0.21]	0.85 [0.84, 0.87]	0.72 [0.70, 0.74]
2000	multi-class VIM - discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.09 [0.08, 0.10]	0.57 [0.55, 0.59]
2000	multi-class VIM - discr. VIM (wsquared_wogini)	1.00 [0.99, 1.00]	0.72 [0.70, 0.74]	0.38 [0.36, 0.39]
2000	multi-class VIM - discr. VIM (wosquared_wogini)	0.01 [0.00, 0.01]	0.02 [0.01, 0.02]	0.04 [0.03, 0.05]

Table S7: Mean AUC values with 95% confidence intervals per considered sample size and method for $C = 10$: class-associated covariates vs. X_{two_gr}.

n	method	$X_{cl\_as\_1}$	$X_{cl\_as\_2}$	$X_{cl\_as\_3}$	$X_{cl\_as\_4}$
100	perm	0.05 [0.04, 0.06]	0.40 [0.38, 0.43]	0.44 [0.41, 0.46]	0.74 [0.72, 0.76]
100	gini_corr	0.03 [0.02, 0.04]	0.30 [0.28, 0.32]	0.41 [0.38, 0.43]	0.77 [0.75, 0.79]
100	discr. VIM (wsquared_wgini)	0.02 [0.02, 0.03]	0.35 [0.33, 0.37]	0.44 [0.42, 0.46]	0.77 [0.75, 0.79]
100	discr. VIM (wosquared_wgini)	0.02 [0.02, 0.03]	0.35 [0.33, 0.37]	0.44 [0.41, 0.46]	0.77 [0.75, 0.79]
100	discr. VIM (wsquared_wogini)	0.02 [0.01, 0.03]	0.33 [0.31, 0.35]	0.40 [0.38, 0.42]	0.73 [0.71, 0.75]
100	discr. VIM (wosquared_wogini)	0.09 [0.08, 0.10]	0.62 [0.60, 0.65]	0.67 [0.65, 0.69]	0.86 [0.84, 0.87]
100	multi-class VIM (wsquared_wgini)	0.21 [0.19, 0.23]	0.63 [0.61, 0.65]	0.66 [0.64, 0.68]	0.86 [0.84, 0.87]
100	multi-class VIM (wosquared_wgini)	0.15 [0.14, 0.17]	0.53 [0.50, 0.55]	0.60 [0.58, 0.62]	0.85 [0.83, 0.86]
100	multi-class VIM (wsquared_wogini)	0.21 [0.19, 0.22]	0.62 [0.60, 0.64]	0.66 [0.64, 0.68]	0.86 [0.84, 0.87]
100	multi-class VIM (wosquared_wogini)	0.13 [0.12, 0.15]	0.49 [0.47, 0.51]	0.58 [0.55, 0.60]	0.84 [0.82, 0.86]
100	multi-class VIM - discr. VIM (wsquared_wgini)	0.73 [0.71, 0.75]	0.75 [0.73, 0.77]	0.71 [0.70, 0.73]	0.70 [0.68, 0.72]
100	multi-class VIM - discr. VIM (wosquared_wgini)	0.49 [0.46, 0.51]	0.61 [0.59, 0.63]	0.65 [0.63, 0.67]	0.79 [0.77, 0.80]
100	multi-class VIM - discr. VIM (wsquared_wogini)	0.93 [0.92, 0.94]	0.78 [0.76, 0.79]	0.71 [0.69, 0.73]	0.53 [0.51, 0.55]
100	multi-class VIM - discr. VIM (wosquared_wogini)	0.61 [0.58, 0.63]	0.35 [0.33, 0.37]	0.38 [0.36, 0.41]	0.40 [0.38, 0.42]
500	perm	0.00 [0.00, 0.00]	0.35 [0.33, 0.37]	0.35 [0.34, 0.37]	0.95 [0.94, 0.95]
500	gini_corr	0.00 [0.00, 0.00]	0.16 [0.15, 0.18]	0.26 [0.24, 0.28]	0.95 [0.94, 0.96]
500	discr. VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.40 [0.38, 0.42]	0.43 [0.41, 0.45]	0.96 [0.95, 0.97]
500	discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.43 [0.41, 0.45]	0.45 [0.43, 0.47]	0.96 [0.96, 0.97]
500	discr. VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.23 [0.21, 0.25]	0.21 [0.19, 0.23]	0.85 [0.83, 0.86]
500	discr. VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.45 [0.43, 0.47]	0.48 [0.46, 0.50]	0.79 [0.77, 0.81]
500	multi-class VIM (wsquared_wgini)	0.10 [0.09, 0.12]	0.96 [0.95, 0.96]	0.94 [0.94, 0.95]	1.00 [1.00, 1.00]
500	multi-class VIM (wosquared_wgini)	0.01 [0.00, 0.01]	0.69 [0.67, 0.71]	0.84 [0.82, 0.85]	1.00 [1.00, 1.00]
500	multi-class VIM (wsquared_wogini)	0.12 [0.10, 0.13]	0.96 [0.95, 0.97]	0.94 [0.94, 0.95]	1.00 [1.00, 1.00]
500	multi-class VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.58 [0.56, 0.60]	0.78 [0.76, 0.79]	1.00 [1.00, 1.00]
500	multi-class VIM - discr. VIM (wsquared_wgini)	0.98 [0.97, 0.98]	0.98 [0.98, 0.99]	0.96 [0.96, 0.97]	0.91 [0.89, 0.92]
500	multi-class VIM - discr. VIM (wosquared_wgini)	0.33 [0.31, 0.35]	0.77 [0.75, 0.79]	0.89 [0.88, 0.90]	0.99 [0.99, 0.99]
500	multi-class VIM - discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	0.97 [0.97, 0.98]	0.97 [0.96, 0.98]	0.73 [0.71, 0.75]
500	multi-class VIM - discr. VIM (wosquared_wogini)	0.99 [0.98, 0.99]	0.61 [0.59, 0.64]	0.70 [0.68, 0.72]	0.86 [0.84, 0.87]
1000	perm	0.00 [0.00, 0.00]	0.34 [0.32, 0.36]	0.29 [0.28, 0.31]	0.98 [0.97, 0.98]
1000	gini_corr	0.00 [0.00, 0.00]	0.11 [0.10, 0.12]	0.17 [0.16, 0.19]	0.99 [0.98, 0.99]
1000	discr. VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.41 [0.39, 0.43]	0.37 [0.35, 0.39]	0.99 [0.99, 0.99]
1000	discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.45 [0.43, 0.47]	0.40 [0.38, 0.42]	0.99 [0.99, 0.99]
1000	discr. VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.18 [0.16, 0.19]	0.11 [0.10, 0.13]	0.89 [0.88, 0.90]
1000	discr. VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.29 [0.28, 0.31]	0.34 [0.32, 0.36]	0.78 [0.76, 0.80]
1000	multi-class VIM (wsquared_wgini)	0.08 [0.07, 0.09]	0.99 [0.99, 1.00]	0.99 [0.99, 0.99]	1.00 [1.00, 1.00]
1000	multi-class VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.79 [0.78, 0.81]	0.94 [0.93, 0.95]	1.00 [1.00, 1.00]
1000	multi-class VIM (wsquared_wogini)	0.10 [0.09, 0.11]	1.00 [0.99, 1.00]	0.99 [0.99, 1.00]	1.00 [1.00, 1.00]
1000	multi-class VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.65 [0.63, 0.67]	0.89 [0.88, 0.90]	1.00 [1.00, 1.00]
1000	multi-class VIM - discr. VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [0.99, 1.00]	0.97 [0.97, 0.98]
1000	multi-class VIM - discr. VIM (wosquared_wgini)	0.33 [0.31, 0.35]	0.86 [0.85, 0.88]	0.98 [0.97, 0.98]	1.00 [1.00, 1.00]
1000	multi-class VIM - discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	0.99 [0.99, 1.00]	1.00 [0.99, 1.00]	0.83 [0.82, 0.84]
1000	multi-class VIM - discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	0.80 [0.78, 0.81]	0.85 [0.84, 0.87]	0.98 [0.97, 0.98]
2000	perm	0.00 [0.00, 0.00]	0.29 [0.27, 0.31]	0.22 [0.21, 0.24]	1.00 [1.00, 1.00]
2000	gini_corr	0.00 [0.00, 0.00]	0.06 [0.05, 0.07]	0.10 [0.09, 0.11]	1.00 [1.00, 1.00]
2000	discr. VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.36 [0.34, 0.38]	0.31 [0.29, 0.33]	1.00 [1.00, 1.00]
2000	discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.42 [0.40, 0.44]	0.34 [0.32, 0.35]	1.00 [1.00, 1.00]
2000	discr. VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.10 [0.09, 0.11]	0.04 [0.03, 0.05]	0.91 [0.90, 0.92]
2000	discr. VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.10 [0.09, 0.11]	0.18 [0.17, 0.20]	0.72 [0.70, 0.74]
2000	multi-class VIM (wsquared_wgini)	0.05 [0.04, 0.06]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.87 [0.86, 0.89]	0.99 [0.98, 0.99]	1.00 [1.00, 1.00]
2000	multi-class VIM (wsquared_wogini)	0.08 [0.07, 0.09]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.71 [0.69, 0.73]	0.97 [0.96, 0.98]	1.00 [1.00, 1.00]
2000	multi-class VIM - discr. VIM (wsquared_wgini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [0.99, 1.00]
2000	multi-class VIM - discr. VIM (wosquared_wgini)	0.28 [0.26, 0.30]	0.95 [0.94, 0.96]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]
2000	multi-class VIM - discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	1.00 [1.00, 1.00]	0.92 [0.91, 0.93]
2000	multi-class VIM - discr. VIM (wosquared_wogini)	1.00 [1.00, 1.00]	0.95 [0.95, 0.96]	0.97 [0.96, 0.98]	1.00 [1.00, 1.00]

Table S8: Mean AUC values with 95% confidence intervals per considered sample size and method for $C = 10$: class-associated covariates vs. X_{thr_gr}.

n	method	$X_{cl\_as\_1}$	$X_{cl\_as\_2}$	$X_{cl\_as\_3}$	$X_{cl\_as\_4}$
100	perm	0.04 [0.03, 0.05]	0.36 [0.33, 0.38]	0.38 [0.36, 0.41]	0.72 [0.71, 0.74]
100	gini_corr	0.02 [0.01, 0.02]	0.25 [0.23, 0.27]	0.34 [0.32, 0.36]	0.72 [0.70, 0.74]
100	discr. VIM (wsquared_wgini)	0.01 [0.01, 0.02]	0.29 [0.27, 0.31]	0.38 [0.36, 0.40]	0.74 [0.72, 0.76]
100	discr. VIM (wosquared_wgini)	0.02 [0.01, 0.02]	0.30 [0.27, 0.32]	0.38 [0.35, 0.40]	0.74 [0.72, 0.76]
100	discr. VIM (wsquared_wogini)	0.02 [0.01, 0.02]	0.29 [0.27, 0.32]	0.37 [0.35, 0.39]	0.73 [0.71, 0.75]
100	discr. VIM (wosquared_wogini)	0.05 [0.04, 0.06]	0.51 [0.49, 0.54]	0.56 [0.53, 0.58]	0.79 [0.78, 0.81]
100	multi-class VIM (wsquared_wgini)	0.14 [0.12, 0.15]	0.52 [0.50, 0.54]	0.54 [0.52, 0.57]	0.78 [0.76, 0.80]
100	multi-class VIM (wosquared_wgini)	0.09 [0.08, 0.11]	0.41 [0.39, 0.43]	0.48 [0.46, 0.50]	0.77 [0.75, 0.79]
100	multi-class VIM (wsquared_wogini)	0.14 [0.12, 0.15]	0.51 [0.49, 0.53]	0.54 [0.52, 0.57]	0.78 [0.76, 0.80]
100	multi-class VIM (wosquared_wogini)	0.08 [0.07, 0.09]	0.37 [0.35, 0.40]	0.45 [0.43, 0.47]	0.75 [0.74, 0.77]
100	multi-class VIM - discr. VIM (wsquared_wgini)	0.62 [0.60, 0.64]	0.68 [0.66, 0.70]	0.64 [0.62, 0.66]	0.65 [0.62, 0.67]
100	multi-class VIM - discr. VIM (wosquared_wgini)	0.35 [0.33, 0.38]	0.51 [0.49, 0.53]	0.55 [0.52, 0.57]	0.70 [0.68, 0.72]
100	multi-class VIM - discr. VIM (wsquared_wogini)	0.90 [0.88, 0.91]	0.74 [0.72, 0.76]	0.67 [0.65, 0.69]	0.49 [0.47, 0.52]
100	multi-class VIM - discr. VIM (wosquared_wogini)	0.61 [0.59, 0.63]	0.39 [0.36, 0.41]	0.40 [0.37, 0.42]	0.41 [0.39, 0.44]
500	perm	0.00 [0.00, 0.00]	0.26 [0.25, 0.28]	0.26 [0.24, 0.28]	0.90 [0.89, 0.91]
500	gini_corr	0.00 [0.00, 0.00]	0.08 [0.07, 0.09]	0.13 [0.12, 0.14]	0.88 [0.87, 0.89]
500	discr. VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.33 [0.31, 0.35]	0.36 [0.33, 0.38]	0.94 [0.93, 0.95]
500	discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.35 [0.33, 0.37]	0.37 [0.34, 0.39]	0.95 [0.94, 0.95]
500	discr. VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.33 [0.31, 0.35]	0.30 [0.28, 0.32]	0.92 [0.91, 0.93]
500	discr. VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.53 [0.51, 0.55]	0.55 [0.53, 0.57]	0.87 [0.86, 0.89]
500	multi-class VIM (wsquared_wgini)	0.02 [0.01, 0.02]	0.80 [0.79, 0.82]	0.77 [0.75, 0.79]	0.98 [0.98, 0.99]
500	multi-class VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.29 [0.27, 0.31]	0.50 [0.48, 0.52]	0.97 [0.96, 0.97]
500	multi-class VIM (wsquared_wogini)	0.02 [0.01, 0.02]	0.79 [0.78, 0.81]	0.77 [0.75, 0.79]	0.98 [0.98, 0.99]
500	multi-class VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.20 [0.18, 0.22]	0.41 [0.39, 0.43]	0.96 [0.95, 0.97]
500	multi-class VIM - discr. VIM (wsquared_wgini)	0.84 [0.82, 0.85]	0.93 [0.92, 0.94]	0.88 [0.87, 0.90]	0.82 [0.80, 0.83]
500	multi-class VIM - discr. VIM (wosquared_wgini)	0.04 [0.03, 0.05]	0.33 [0.31, 0.35]	0.55 [0.53, 0.58]	0.91 [0.90, 0.93]
500	multi-class VIM - discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	0.90 [0.89, 0.91]	0.88 [0.86, 0.89]	0.48 [0.46, 0.50]
500	multi-class VIM - discr. VIM (wosquared_wogini)	0.87 [0.85, 0.88]	0.24 [0.22, 0.26]	0.38 [0.35, 0.40]	0.62 [0.60, 0.64]
1000	perm	0.00 [0.00, 0.00]	0.23 [0.21, 0.25]	0.19 [0.17, 0.20]	0.96 [0.95, 0.96]
1000	gini_corr	0.00 [0.00, 0.00]	0.03 [0.03, 0.04]	0.06 [0.05, 0.07]	0.94 [0.93, 0.95]
1000	discr. VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.33 [0.31, 0.35]	0.30 [0.28, 0.32]	0.98 [0.98, 0.99]
1000	discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.37 [0.35, 0.39]	0.32 [0.30, 0.34]	0.98 [0.98, 0.99]
1000	discr. VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.38 [0.35, 0.40]	0.26 [0.24, 0.28]	0.98 [0.97, 0.98]
1000	discr. VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.48 [0.46, 0.50]	0.52 [0.50, 0.54]	0.92 [0.91, 0.93]
1000	multi-class VIM (wsquared_wgini)	0.00 [0.00, 0.01]	0.92 [0.91, 0.93]	0.88 [0.87, 0.89]	1.00 [1.00, 1.00]
1000	multi-class VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.21 [0.19, 0.22]	0.49 [0.47, 0.51]	1.00 [0.99, 1.00]
1000	multi-class VIM (wsquared_wogini)	0.01 [0.00, 0.01]	0.91 [0.90, 0.92]	0.87 [0.86, 0.89]	1.00 [1.00, 1.00]
1000	multi-class VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.11 [0.10, 0.12]	0.36 [0.34, 0.38]	0.99 [0.99, 1.00]
1000	multi-class VIM - discr. VIM (wsquared_wgini)	0.94 [0.93, 0.95]	0.98 [0.97, 0.98]	0.96 [0.96, 0.97]	0.91 [0.90, 0.92]
1000	multi-class VIM - discr. VIM (wosquared_wgini)	0.00 [0.00, 0.01]	0.22 [0.20, 0.24]	0.59 [0.57, 0.61]	0.98 [0.97, 0.98]
1000	multi-class VIM - discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	0.91 [0.90, 0.92]	0.93 [0.92, 0.94]	0.45 [0.43, 0.47]
1000	multi-class VIM - discr. VIM (wosquared_wogini)	0.89 [0.88, 0.91]	0.19 [0.18, 0.21]	0.38 [0.36, 0.40]	0.73 [0.72, 0.75]
2000	perm	0.00 [0.00, 0.00]	0.17 [0.16, 0.18]	0.13 [0.11, 0.14]	0.99 [0.98, 0.99]
2000	gini_corr	0.00 [0.00, 0.00]	0.01 [0.01, 0.01]	0.01 [0.01, 0.02]	0.98 [0.97, 0.98]
2000	discr. VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.29 [0.27, 0.31]	0.25 [0.23, 0.27]	1.00 [1.00, 1.00]
2000	discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.34 [0.32, 0.36]	0.28 [0.26, 0.30]	1.00 [1.00, 1.00]
2000	discr. VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.42 [0.40, 0.44]	0.23 [0.21, 0.24]	1.00 [1.00, 1.00]
2000	discr. VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.34 [0.32, 0.36]	0.44 [0.42, 0.46]	0.95 [0.94, 0.96]
2000	multi-class VIM (wsquared_wgini)	0.00 [0.00, 0.00]	0.98 [0.97, 0.98]	0.95 [0.94, 0.96]	1.00 [1.00, 1.00]
2000	multi-class VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.12 [0.11, 0.13]	0.52 [0.50, 0.54]	1.00 [1.00, 1.00]
2000	multi-class VIM (wsquared_wogini)	0.00 [0.00, 0.00]	0.97 [0.97, 0.98]	0.94 [0.93, 0.94]	1.00 [1.00, 1.00]
2000	multi-class VIM (wosquared_wogini)	0.00 [0.00, 0.00]	0.04 [0.03, 0.04]	0.34 [0.32, 0.36]	1.00 [1.00, 1.00]
2000	multi-class VIM - discr. VIM (wsquared_wgini)	0.99 [0.98, 0.99]	1.00 [1.00, 1.00]	0.99 [0.99, 0.99]	0.97 [0.96, 0.97]
2000	multi-class VIM - discr. VIM (wosquared_wgini)	0.00 [0.00, 0.00]	0.12 [0.10, 0.13]	0.64 [0.62, 0.66]	1.00 [0.99, 1.00]
2000	multi-class VIM - discr. VIM (wsquared_wogini)	1.00 [1.00, 1.00]	0.94 [0.93, 0.95]	0.96 [0.96, 0.97]	0.40 [0.39, 0.42]
2000	multi-class VIM - discr. VIM (wosquared_wogini)	0.88 [0.86, 0.89]	0.20 [0.18, 0.21]	0.43 [0.41, 0.45]	0.82 [0.80, 0.83]

B Real data analysis

B.1 Overview of the datasets

Table S9: Overview of datasets – I. The following information is provided: "data.id": OpenML ID of the datasets, is "–" for datasets from PMLB, "label": dataset label, "n": sample size, "p": number of covariates, "n_cl": number of outcome classes, "prop_cat": percentage of categorical covariates

data.id	label	n	p	n_cl	prop_cat
1557	abalone	4177	8	3	0.125
–	allhyper	3771	27	4	0.778
458	analcatdata_authorship	841	70	4	0.043
462	analcatdata_challenger	23	3	3	0.667
469	analcatdata_dmft	797	4	6	1.000
475	analcatdata_germangss	400	5	4	0.800
40709	analcatdata_happiness	60	3	3	0.667
–	ann_thyroid	7200	21	3	0.714
1459	artificial-characters	10218	7	10	0.143
–	auto	202	25	5	0.360
1548	autoUniv-au4-2500	2500	100	3	0.650
1555	autoUniv-au6-1000	1000	40	8	0.075
1551	autoUniv-au6-400	400	40	8	0.075
1549	autoUniv-au6-750	750	40	8	0.075
1552	autoUniv-au7-1100	1100	12	5	0.583
1554	autoUniv-au7-500	500	12	5	0.583
1553	autoUniv-au7-700	700	12	3	0.583
11	balance-scale	625	4	3	1.000
1465	breast-tissue	106	9	6	0.000
40663	calendarDOW	399	32	5	0.625
40975	car	1728	6	4	1.000
40664	car-evaluation	1728	21	4	1.000
1466	cardiotocography	2126	35	10	0.400
40700	cars1	392	7	3	0.143
–	cleveland	303	13	5	0.615
40711	cleveland-nominal	303	7	5	1.000
–	cloud	108	7	4	0.143
23	cmc	1473	9	3	0.778
–	collins	485	22	13	0.000
468	confidence	72	3	6	0.000
–	contraceptive	1473	9	3	0.778
41701	CPMP-2015-classification	527	23	4	0.217
41919	CPMP-2015-runtime-classification	527	22	4	0.227
–	dermatology	366	34	6	0.971
1117	desharnais	81	10	3	0.300

Table S10: Overview of datasets – II. The following information is provided: "data.id": OpenML ID of the datasets, is "−" for datasets from PMLB, "label": dataset label, "n": sample size, "p": number of covariates, "n_cl": number of outcome classes, "prop_cat": percentage of categorical covariates

data.id	label	n	p	n_cl	prop_cat
694	diggle_table_a2	310	8	9	0.000
40670	dna	3186	180	3	1.000
39	ecoli	336	7	8	0.286
4340	Engine1	383	5	3	0.000
1044	eye_movements	10936	27	3	0.333
1475	first-order-theorem-proving	6118	51	6	0.000
477	fl2000	67	15	5	0.200
285	flags	194	28	8	0.857
1476	gas-drift	13910	128	6	0.000
1477	gas-drift-different-concentrations	13910	129	6	0.000
4538	GesturePhaseSegmentationProcessed	9873	32	5	0.000
41	glass	214	9	6	0.000
338	grub-damage	155	8	4	0.875
329	hayes-roth	160	4	3	1.000
1565	heart-h	294	13	5	0.692
1512	heart-long-beach	200	13	5	0.615
1513	heart-switzerland	123	12	5	0.667
41972	Indian_pines	9144	220	8	0.000
44344	Iris	150	4	3	0.000
41004	jungle_chess_2pcs_endgame_lion_elephant	4704	44	3	0.727
41000	jungle_chess_2pcs_endgame_panther_elephant	4704	42	3	0.667
40997	jungle_chess_2pcs_endgame_panther_lion	4704	44	3	0.727
40496	LED-display-domain-7digit	500	7	10	1.000
40677	led24	3200	24	10	1.000
40678	led7	3200	7	10	1.000
10	lymph	148	18	4	1.000
277	meta_ensembles.arff	74	62	4	0.145
12	mfeat-factors	2000	216	10	0.000
14	mfeat-fourier	2000	76	10	0.000
16	mfeat-karhunen	2000	64	10	0.000
18	mfeat-morphological	2000	6	10	0.500
40979	mfeat-pixel	2000	240	10	1.000
22	mfeat-zernike	2000	47	10	0.000
41703	MIP-2016-classification	218	116	5	0.095
−	movement_libras	360	90	15	0.000
−	new_thyroid	215	5	3	0.000
26	nursery	12960	8	5	1.000
28	optdigits	5620	62	10	0.177
30	page-blocks	5473	10	5	0.000
339	pasture	36	21	3	0.095
32	pendigits	10992	16	10	0.000
−	penguins	333	7	3	0.429
1100	PopularKids	478	10	3	1.000

Table S11: Overview of datasets – III. The following information is provided: "data.id": OpenML ID of the datasets, is "−" for datasets from PMLB, "label": dataset label, "n": sample size, "p": number of covariates, "n_cl": number of outcome classes, "prop_cat": percentage of categorical covariates

data.id	label	n	p	n_cl	prop_cat
457	prnn_cushings	27	2	4	0.000
952	prnn_fglass	214	9	6	0.000
480	prnn_viruses	61	18	4	0.444
679	rmftsa_sleepdata	1024	2	4	0.000
1516	robot-failures-lp1	88	90	4	0.000
1517	robot-failures-lp2	47	90	5	0.000
1518	robot-failures-lp3	47	90	4	0.000
1519	robot-failures-lp4	117	90	3	0.000
1520	robot-failures-lp5	164	90	5	0.000
182	satimage	6430	36	6	0.000
−	schizo	340	14	3	0.143
1499	seeds	210	7	3	0.000
40984	segment	2310	18	7	0.111
1500	seismic-bumps	210	7	3	0.000
1501	semeion	1593	256	10	1.000
4153	Smartphone-Based_Recognition_of_Human_Activities	180	66	6	0.000
40686	solar-flare	315	12	5	1.000
−	solar_flare_2	1066	9	6	1.000
−	soybean	675	35	18	1.000
46	splice	3190	60	3	1.000
40982	steel-plates-fault	1941	27	7	0.111
1115	teachingAssistant	151	5	3	0.400
40499	texture	5500	40	11	0.000
40682	thyroid-new	215	5	3	0.000
45492	timing-attack-dataset-15-micro-seconds-delay-2022-09-18	9995	16	11	0.000
42544	Touch2	265	10	8	0.100
1508	user-knowledge	403	5	5	0.000
1047	usp05	197	12	10	0.417
41082	USPS	9298	256	10	0.000
54	vehicle	846	18	4	0.000
1523	vertebra-column	310	6	3	0.000
685	visualizing_livestock	130	2	5	0.000
307	vowel	990	12	11	0.083
1497	wall-robot-navigation	5456	24	4	0.000
−	waveform_21	5000	21	3	0.000
−	waveform_40	5000	40	3	0.000
343	white-clover	63	31	4	0.129
187	wine	178	13	3	0.000
40691	wine-quality-red	1599	11	6	0.000
40498	wine-quality-white	4898	11	7	0.000
−	wine_recognition	178	13	3	0.000
181	yeast	1484	8	10	0.250
62	zoo	101	16	7	1.000

B.2 Dataset-specific performance metric values: `wsquared_wgini` versus conventional RFs

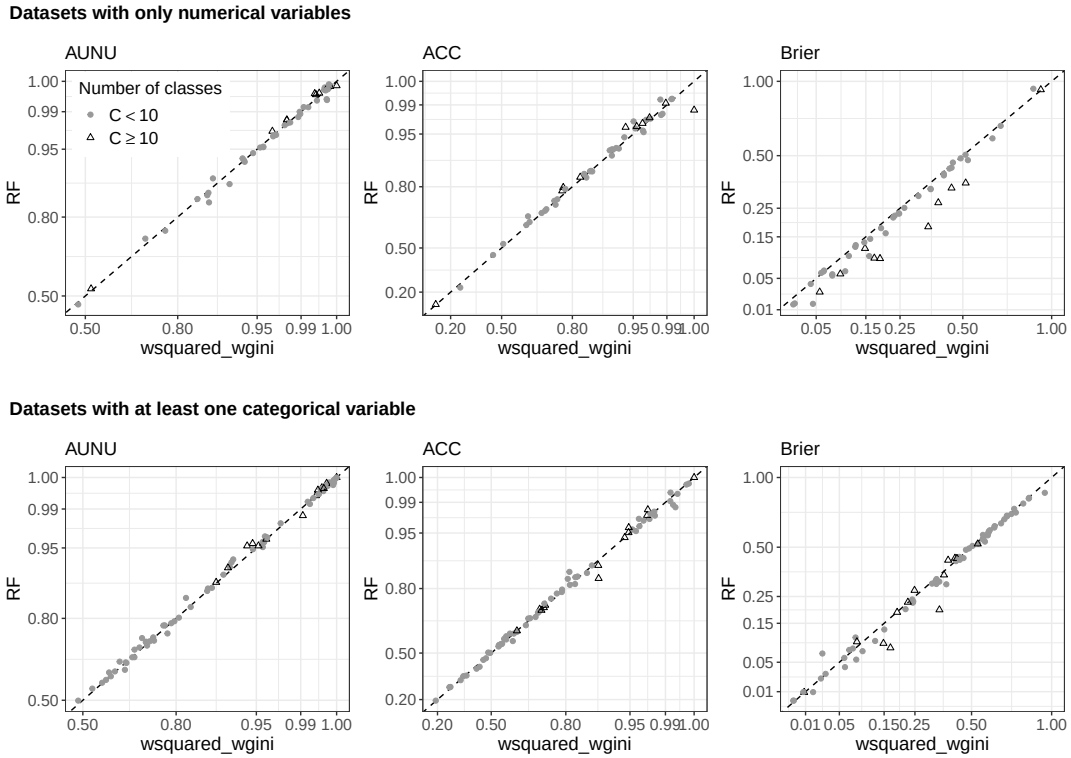

Fig. S15: Dataset-specific performance metric values: `wsquared_wgini` versus `RF`. The white diamonds mark the datasets that have ten or more outcome classes. The dashed lines represent the diagonal. Axes were transformed for visual clarity: A negative complementary square root transformation was applied to AUNU and ACC, and a square root transformation was used for Brier. This is a variant of Figure 6 from the main paper, distinguishing between datasets that contain only numerical covariates and those that also include categorical covariates.

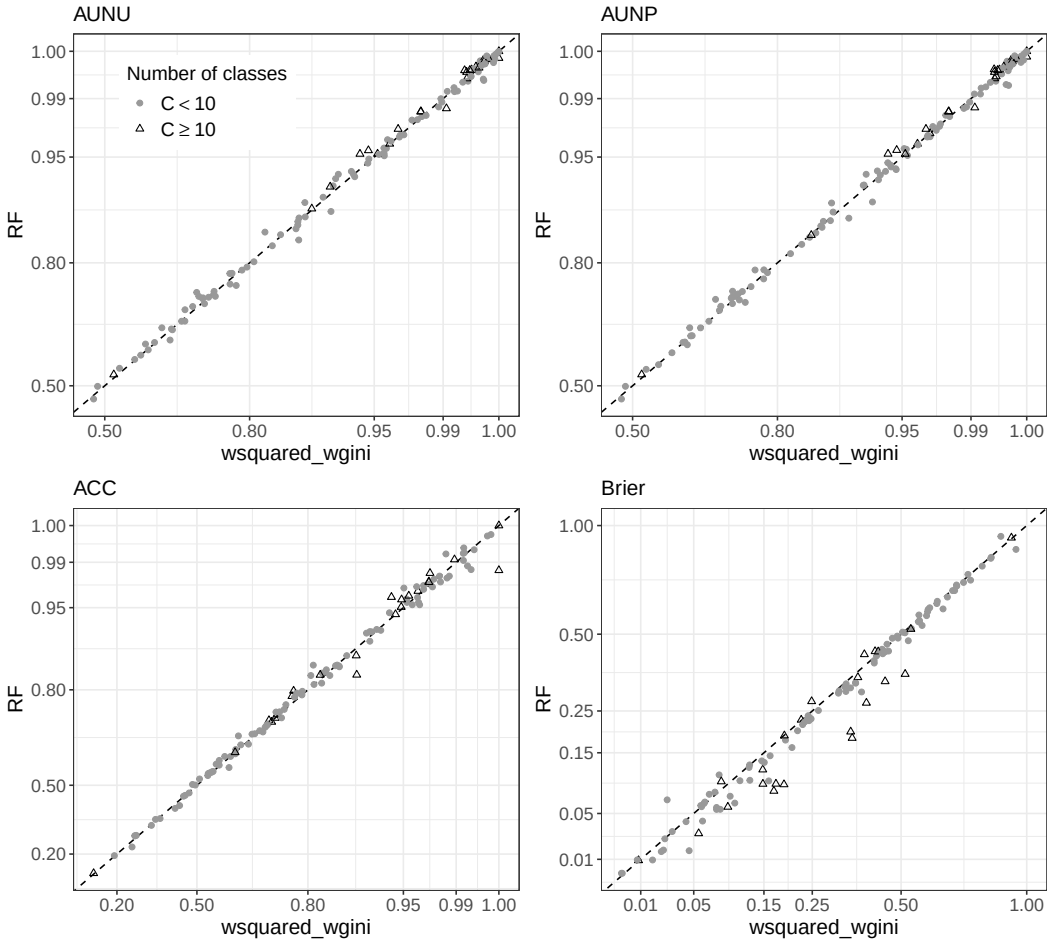

Fig. S16: Dataset-specific performance metric values: `wsquared_wgini` versus `RF`. The white diamonds mark the datasets that have ten or more outcome classes. The dashed lines represent the diagonal. Axes were transformed for visual clarity: A negative complementary square root transformation was applied to AUNU, AUNP and ACC, and a square root transformation was used for Brier. This is a variant of Figure 6 from the main paper that includes AUNP.